AP Calculus AB
Chapter 5 Review

Name Key

1. Find $\frac{d}{dx}\int_2^x (1-t^3)dt$

$(1-(x)^3)\cdot 1$

$1-x^3$

2. Find $\frac{d}{dx}\int_2^{x^2} (1-t^3)dt$

$(1-(x^2)^3)\cdot 2x$

$(1-x^6)2x$

$2x-2x^7$

3. Evaluate the Riemann Sum for $f(x)=x^3-x$ on the interval $0 \le x \le 2$ with four subintervals.

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$ $x_0=0$ $x_1=.5$ $x_2=1$ $x_3=1.5$ $x_4=2$

$R_4 = (\frac{1}{2})[f(.5)+f(1)+f(1.5)+f(2)]$

$= 3.75$

4. Approximate $\int_2^4 (3x^2-2x)dx$ using

a) Midpoint rule with 4 subintervals
b) Trapezoid rule with 4 subintervals

$\Delta x = \frac{4-2}{4} = \frac{1}{2}$

$x_0=2$ $x_1=2.5$ $x_2=3$ $x_3=3.5$ $x_4=4$

Are each of these values overestimates or underestimates of the actual area?

a) midpoints: 2.25, 2.75, 3.25, 3.75

$M_4 = (\frac{1}{2})[f(2.25)+f(2.75)+f(3.25)+f(3.75)]$

$= 43.875$ → underestimate

b) $T_4 = (\frac{1}{2})(\frac{1}{2})[f(2)+2f(2.5)+2f(3)+2f(3.5)+f(4)]$

$= 44.25$ → overestimate

Actual Area: 44

5. A particle's velocity, given in feet per second, is described by the equation $v(t) = 3t^2 - 12t + 9$ when $0 \le t \le 4$. Find:

a) the particle's total distance traveled on the given time interval
b) the particle's displacement after 4 seconds

a) $0 = 3t^2 - 12t + 9$
$0 = 3(t^2 - 4t + 3)$
$0 = 3(t-3)(t-1)$
$t = 1, 3$

$\left|\int_0^1 v(t)dt\right| + \left|\int_1^3 v(t)dt\right| + \left|\int_3^4 v(t)dt\right|$

$4 + |-4| + 4$

12 ft.

b) $\int_0^4 v(t)dt$

4 ft.

6. Given: $a(t) = 12t^2 - 2$, $v(2) = 25$, and $s(2) = 4$, write an expression for $s(t)$

$v(t) = \int 12t^2 - 2\, dt$
$v(t) = 4t^3 - 2t + C$
$(25) = 4(2)^3 - 2(2) + C$
$25 = 32 - 4 + C$
$25 = 28 + C$
$-28 \quad -28$
$C = -3$

$v(t) = 4t^3 - 2t - 3$
$s(t) = \int 4t^3 - 2t - 3\, dt$
$s(t) = t^4 - t^2 - 3t + C$
$(4) = (2)^4 - (2)^2 - 3(2) + C$
$4 = 16 - 4 - 6 + C$
$4 = 6 + C$
$-2 = C$

$s(t) = t^4 - t^2 - 3t - 2$

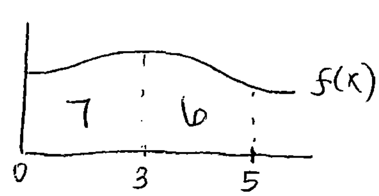
7. Suppose $\int_0^3 f(x)dx = 7$ and $\int_0^5 f(x)dx = 13$, find:

a) $\int_3^5 f(x)dx = 6$

b) $\int_0^3 4f(x)dx = 28$

c) $\int_5^0 f(x)dx = -13$

d) $\int_3^3 f(x)dx = 0$



8. Solve the differential equation

$$\frac{dy}{dx} = x\sqrt{\frac{y}{x^2+1}}$$

$$\frac{dy}{dx} = \frac{x\sqrt{y}}{\sqrt{x^2+1}}$$

$$\sqrt{x^2+1}\,dy = x\sqrt{y}\,dx$$

$$\int \frac{1}{\sqrt{y}}\,dy = \int \frac{x}{\sqrt{x^2+1}}\,dx$$

$$\int y^{-1/2}dy$$

$$2y^{1/2}$$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$$\int \frac{\cancel{x}}{u^{1/2}} \cdot \frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int u^{-1/2}\,du$$

$$u^{1/2} + C$$

$$2y^{1/2} = (x^2+1)^{1/2} + C$$

Evaluate each of the following integrals

9. $\int x\sqrt{x+2}dx$

$u = x+2$

$\hookrightarrow x = u-2$

$du = dx$

$$\int x \cdot u^{1/2}\,du$$

$$\int (u-2)u^{1/2}du$$

$$\int u^{3/2} - 2u^{1/2}\,du$$

$$\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$

$$\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

10. $\int \sin 2x\sqrt{\cos 2x - 3}dx$

$u = \cos 2x - 3$

$\frac{du}{dx} = -\sin 2x \cdot 2$

$dx = \frac{du}{-2\sin 2x}$

$$\int \cancel{\sin 2x} \cdot u^{1/2} \cdot \frac{du}{-2\cancel{\sin 2x}}$$

$$-\frac{1}{2}\int u^{1/2}\,du$$

$$-\frac{1}{3}u^{3/2} + C$$

$$-\frac{1}{3}(\cos 2x - 3)^{3/2} + C$$

11. $\int x^2 - 3x + \frac{1}{x} + \frac{2}{x^2}dx$

$$\frac{1}{3}x^3 - \frac{3}{2}x^2 + \ln|x| + -\frac{2}{x} + C$$

12. $\int \frac{x^4 + x^2 - 3}{x^2}dx$

$$\int (x^4 + x^2 - 3)x^{-2}dx$$

$$\int x^2 + x^0 - 3x^{-2}\,dx$$

$$\int x^2 + 1 - 3x^{-2}dx$$

$$\frac{1}{3}x^3 + x + 3x^{-1} + C$$

$$\frac{1}{3}x^3 + x + \frac{3}{x} + C$$

13. $\int_3^{11} \frac{dx}{\sqrt{2x+3}}$

$u = 2x+3$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int_9^{25} \frac{1}{u^{1/2}} \cdot \frac{du}{2}$

$\frac{1}{2}\int_9^{25} u^{-1/2}\, du$

$u^{1/2} \Big|_9^{25}$

$[(25)^{1/2}] - [(9)^{1/2}]$

$5 - 3$

2

14. $\int \frac{\cos(\ln x)}{x} dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$x\,du = dx$

$\int \frac{\cos u}{\cancel{x}} \cdot \cancel{x}\,du$

$\int \cos u\, du$

$\sin u + C$

$\sin(\ln x) + C$

15. $\int_0^3 \frac{x^3}{x^4+1} dx$

$u = x^4 + 1$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

$\int_1^{82} \frac{\cancel{x^3}}{u} \cdot \frac{du}{4\cancel{x^3}}$

$\frac{1}{4}\int_1^{82} u\, du$

$\frac{1}{4} \ln|u| \Big|_1^{82}$

$\frac{1}{4}[\ln|82| - \ln|1|]$

$\frac{1}{4}\ln 82$

16. $\int_2^5 x^3 - 2x^2 + 3x^{-1} dx$

$\frac{1}{4}x^4 - \frac{2}{3}x^3 + 3\ln|x| \Big|_2^5$

$\left[\frac{1}{4}(5)^4 - \frac{2}{3}(5)^3 + 3\ln 5\right] - \left[\frac{1}{4}(2)^4 - \frac{2}{3}(2)^3 + 3\ln 2\right]$

17. $\int \frac{8^{\sqrt{x}}}{\sqrt{x}} dx$

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x}\,du = dx$

$\int \frac{8^u}{\cancel{\sqrt{x}}} \cdot 2\cancel{\sqrt{x}}\,du$

$2\int 8^u\, du$

$2 \cdot \frac{1}{\ln 8} \cdot 8^u + C$

$\frac{2 \cdot 8^{\sqrt{x}}}{\ln 8} + C$

18. $\int \frac{x+1}{x^2+1} dx$

$\int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u\, du$

$\frac{1}{2}\ln|x^2+1| + \arctan x + C$